Optomechanically Induced Transparency in the Single-Photon Strong Coupling Regime

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Optomechanical systems have been shown both theoretically and experimentally to exhibit an analogon to atomic electromagnetically induced transparency, with sharp transmission features that are controlled by a second laser beam. Here we investigate these effects in the regime where single photons and phonons interact strongly. We demonstrate that pulsed transistor-like switching of transmission still works even in this regime. We show that optomechanically induced transparency at the second mechanical sideband could be a sensitive tool to see first indications of this challenging regime even at moderate coupling strengths.

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Introduction – The field of optomechanics explores the coupling between photons and phonons via radiation pressure. It aims at applications in classical and quantum information processing as well as ultrasensitive measurements and tests of fundamental quantum effects using mesoscopic or macroscopic systems [1–4]. Recently, an interesting feature called "optomechanically induced transparency" (OMIT) has been predicted theoretically [5] and observed experimentally [6, 7]: The photon transmission through an optomechanical cavity is drastically influenced when introducing a second laser beam. Remarkably, the spectral range in which the photon transmission is modified is of the order of the mechanical decay rate, which typically is very small. OMIT can thus be employed for slowing and stopping light or for operating a "transistor", where photon transmission is switched on and off optically [5–8]. OMIT is an analogon of atomic electromagnetically induced transparency [9], where a medium consisting of three-level atoms can be made transparent by illuminating it with a second laser.

In optomechanical systems, the regime where *single* photons couple strongly to *single* phonons has not yet been reached except for cold atom clouds [10–12], but experiments have shown huge progress during the last years [13–16]. This so-called single-photon strong coupling regime has attracted large theoretical interest leading to the prediction of optical Schrödinger cat states [17, 18], a classical to quantum crossover in nonlinear optomechanical systems [19], non-Gaussian [20] and non-classical mechanical states [21–23], as well as multiple cooling resonances [24]. Certain dark states [25], photon antibunching [26, 27], a crossover from sub-Poissonian to super-Poissonian statistics, as well as photon cascades [27] may be observed. Recently, two-mode setups have been shown to be favorable for reaching strong photon and phonon nonlinearities [28, 29].

In this paper, we analyze OMIT in the single-photon

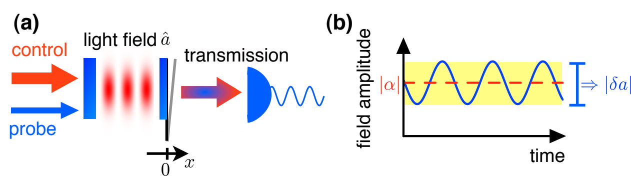

Figure 1: (a) Standard optomechanical setup driven by a control and a probe laser. The transmitted field amplitude will be detected. (b) Transmitted field amplitude as a function of time in a frame rotating at the control laser frequency ω_c. The control laser generates a constant field amplitude α (red, dashed line). The probe laser induces oscillations on top of α at a beat frequency of $\omega_p - \omega_c$. The amplitude of these oscillations, Eq. (6), is our signal (cf. main text).

strong coupling regime. By direct simulations of the full quantum dissipative dynamics, we study both the spectroscopic signal and the time-evolution during pulsed operation. In analyzing higher mechanical sidebands, we find that OMIT may turn out to be a crucial tool to observe first tell-tale effects of the single-photon strong coupling regime even at moderate couplings, in contrast to the standard single-beam situation.

Model – We consider a generic optomechanical system, where an optical cavity is coupled to mechanical motion, cf. Fig. 1(a). The system is described by the Hamiltonian [30]

$$\hat{H} = \hbar\omega_{\rm cav}\hat{a}^\dagger\hat{a} + \hbar\Omega\hat{b}^\dagger\hat{b} - \hbar g_0\left(\hat{b}^\dagger + \hat{b}\right)\hat{a}^\dagger\hat{a} + \hat{H}_{\rm dr}\,, \quad (1)$$

where $\hat{a}$ ($\hat{b}$) is the photon (phonon) annihilation operator, $\omega_{\rm cav}$ is the cavity resonance frequency, Ω the mechanical frequency and g_0 the optomechanical coupling between single photons and phonons. $\hat{H}_{\rm dr}$ describes the two-tone laser driving,

$$\hat{H}_{\rm dr} = \hbar\left[\alpha_c e^{-\mathrm{i}\omega_c t} + \alpha_p e^{-\mathrm{i}\omega_p t}\right]\hat{a}^\dagger + \mathrm{H.c}\,, \quad (2)$$

where ω_c (ω_p) and α_c (α_p) are the control (probe) laser frequency and amplitude, respectively.

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Physical insight can be gained by diagonalizing the optomechanical interaction, introducing a mechanical shift depending on the photon number [17, 18]. Applying the unitary "polaron" transformation $\hat{\tilde{H}} = \hat{U}^\dagger \hat{H} \hat{U}$ to (1), with $\hat{U} = \exp\left[g_0 \hat{a}^\dagger \hat{a}\left(\hat{b}^\dagger - \hat{b}\right)/\Omega\right]$, one finds

$$\hat{\tilde{H}} = \hbar\omega_{\rm cav}^{\rm eff}\hat{a}^\dagger\hat{a} + \hbar\Omega\hat{b}^\dagger\hat{b} - \hbar\frac{g_0^2}{\Omega}\hat{a}^\dagger\hat{a}^\dagger\hat{a}\hat{a} + \hat{\tilde{H}}_{\rm dr}\,, \qquad (3)$$

where we have defined $\omega_{\rm cav}^{\rm eff} = \omega_{\rm cav} - g_0^2/\Omega$. The laser driving has been transformed to

$$\hat{\tilde{H}}_{\rm dr} = \hbar\left[\alpha_c e^{-{\rm i}\omega_c t} + \alpha_p e^{-{\rm i}\omega_p t}\right]\hat{a}^\dagger \exp\left[-\frac{g_0}{\Omega}\left(\hat{b}^\dagger - \hat{b}\right)\right] + {\rm H.c.}\,, \qquad (4)$$

giving rise to Franck-Condon physics [17, 18, 20, 26, 27], cf. Fig. 2(a). We define the control/probe detuning from the effective cavity resonance frequency as $\Delta_{c/p} = \omega_{c/p} - \omega_{\rm cav}^{\rm eff}$. The eigenstates of (3) without laser driving are given by $|n_a, n_b\rangle$, where n_a and n_b denote the number of photons and of phonons (in the shifted frame), respectively.

Dissipative Dynamics – The dissipative dynamics of the optomechanical system can be described by a Lindblad master equation

$$\dot{\hat{\rho}} = \frac{\rm i}{\hbar}\left[\hat{\rho}, \hat{H}\right] + \kappa\mathcal{D}[\hat{a}]\hat{\rho} + \Gamma_M\mathcal{D}[\hat{b}]\hat{\rho}\,. \qquad (5)$$

$\hat{\rho}$ denotes the density matrix for the system comprising the optical and mechanical mode. κ (Γ_M) describes the photon (phonon) decay rate, and $\mathcal{D}\left[\hat{A}\right]\hat{\rho} = \hat{A}\hat{\rho}\hat{A}^\dagger - \hat{A}^\dagger\hat{A}\hat{\rho}/2 - \hat{\rho}\hat{A}^\dagger\hat{A}/2$ is the dissipation superoperator in Lindblad form. Note that we have assumed a zero temperature mechanical bath essentially assuming $k_B T/\hbar\Omega \ll 1$, where $k_B T$ sets the thermal energy of the phonon bath. We solve Eq. (5) in the time-domain numerically, where we use the original optomechanical Hamiltonian (1). This allows us to also consider pulse-based schemes.

Two-tone transmission – In the steady state, the transmitted field amplitude of the two-tone driven optomechanical cavity in a frame rotating at the control frequency ω_c is given by

$$\langle\hat{a}\rangle = \alpha + \delta a_1 e^{-{\rm i}(\omega_p - \omega_c)t} + \delta a_{-1} e^{{\rm i}(\omega_p - \omega_c)t}\,. \qquad (6)$$

Higher harmonics of the beat frequency $\omega_p - \omega_c$ can also occur in principle due to the photon nonlinearity of the polaron-transformed Hamiltonian (3), but they turn out to be weak in our analysis. Note that the second term is neglected typically in a linearized description in the resolved sideband limit. However, in the strong coupling regime, this contribution becomes important due to additional mechanical sidebands, cf. Fig. 2(a). In the following, we are interested in what we term the probe beam transmission $|\delta a|^2 = |\delta a_1|^2 + |\delta a_{-1}|^2$, the most important observable in the context of OMIT. It can be measured

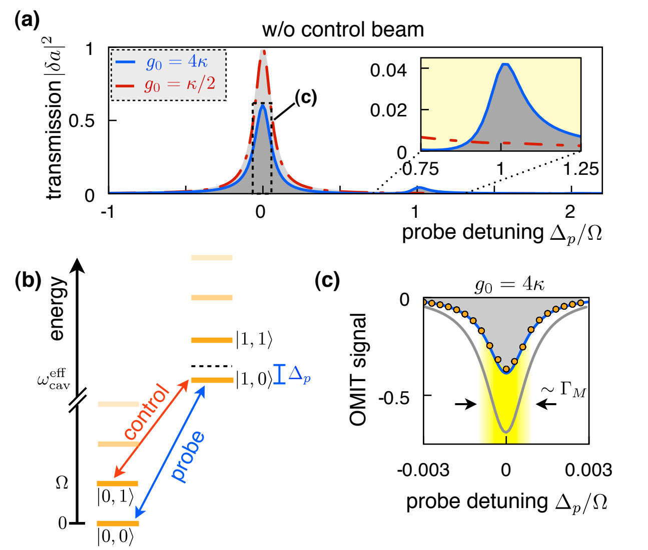

Figure 2: Single-photon strong coupling effects and OMIT. (a) Probe transmission for different optomechanical couplings g_0, at zero control beam amplitude [20], for comparison. Mechanical sidebands clearly appear for $g_0 \gtrsim \kappa$. (b) Energy level scheme of the optomechanical system, see (3). Levels $|n_a, n_b\rangle$ are indexed by the number of photons n_a and phonons n_b. (c) The OMIT signal, i.e. the difference between $|\delta a|^2$ with and without a control beam (normalized to the maximum transmission of a linear cavity). Orange circles: Numerical results for $g_0/\kappa = 4$. Grey line: Expectation for the standard, linearized regime, Eq. (7). Blue line: Expectation of Eq. (7), but with $|\alpha|^2 \rightarrow \langle\hat{a}^\dagger\hat{a}\rangle$ and where Franck-Condon factors are taken into account, see main text. [Parameters: (a) $\alpha_p = 10^{-4}\,\Omega$, $\kappa = \Omega/8$, $\Gamma_M = 10^{-3}\,\Omega$. (c) same as in (a), but $\alpha_c = 10^{-2}\,\Omega$, $\Delta_c = -\Omega$].

by a heterodyne setup [31] by mixing with a local oscillator at ω_c and obtaining the power in the signal at the beat frequency.

Standard prediction – OMIT has so far been studied only in the *linearized* regime of optomechanics, where everything only depends on the product $g_0|\alpha|$ of the coupling and the field amplitude α inside the cavity. For this to be valid, the coupling g_0 has to be much smaller than the photon decay rate κ.

We recall that the most common OMIT signature arises when the control laser drives the cavity on the red sideband, i.e. $\Delta_c = -\Omega$. When analyzing the probe beam transmission as a function of the probe detuning Δ_p, a transmission dip appears on resonance (i.e. $\Delta_p = 0$), cf. Fig. 2(c). The dip's width is $\sim\Gamma_M$, which is typically much smaller than the cavity line width κ. The transmission of the probe beam is given by [5, 32]

$$\delta a = \frac{-{\rm i}\alpha_p}{-{\rm i}\Delta_p + \frac{\kappa}{2} - 2{\rm i}\frac{g_0^2}{\Omega}|\alpha|^2\chi\left[\omega_p - \omega_c\right]}\,, \qquad (7)$$

where $\chi^{-1}[\omega] = 1 - (\omega/\Omega)^2 - {\rm i}\omega\Gamma_M/\Omega^2$ is the (rescaled) mechanical susceptibility.

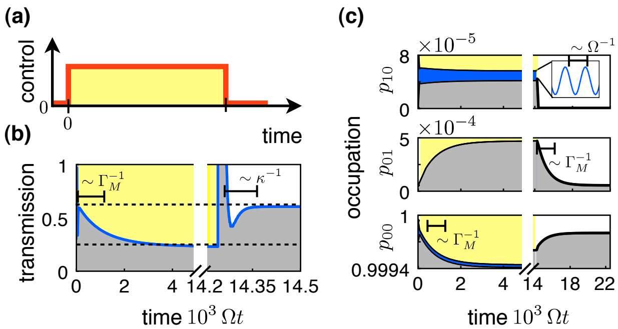

Figure 3: Pulsed switching at the OMIT dip, for $g_0/\kappa = 4$. (a) The probe laser drives the cavity continuously. At time $t = 0$, the control laser is switched on. This leads to a decrease in the probe beam transmission building up on a scale $\sim \Gamma_M^{-1}$, cf. (b). When the control laser is suddenly switched off, the probe transmission increases again on a scale $\sim \kappa^{-1} \ll \Gamma_M^{-1}$. (c) Occupation transfer between individual quantum states induced by the control beam. Oscillations at Ω are clearly visible in p_{10} (i.e. for the state with 1 photon, 0 phonons). [Parameters: same as in Fig 2(c) and $\Delta_p = 0$].

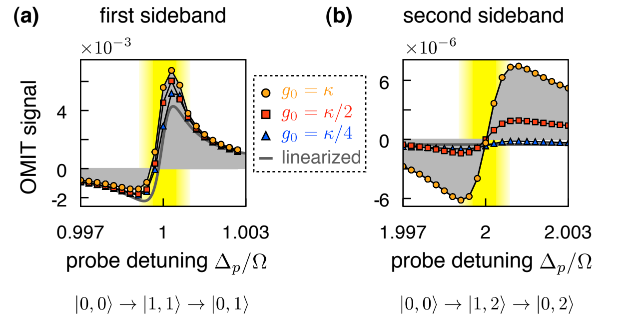

Figure 4: Crossover from moderate coupling to the single-photon strong coupling regime on the first (a) and second (b) sideband. The OMIT signal at the second sideband is a clear signature of single-photon strong coupling physics, since the linearized analysis (valid for $g_0/\kappa \to 0$ at fixed $g_0 |\alpha|$) fails to show an OMIT signature at this sideband. This signature becomes visible even for moderate coupling strengths. The yellow shaded regions depict the width of the OMIT features $\sim \Gamma_M$. Symbols: OMIT signal for different g_0/κ where $g_0 |\alpha|$ and κ are kept fixed. Grey line: expectation for the linearized regime from Eq. (7). The energy levels denote the relevant transitions induced by the probe and control. [Parameters: same as in Fig. 2(c), but $\Delta_c = 0$ and $g_0\alpha_c = 1.25 \cdot 10^{-3}\,\Omega^2 =$ const]

In the vicinity of the transmission dip, the beat frequency $\omega_p - \omega_c$ between probe and control is given by the mechanical frequency Ω. Thus, the mechanical resonator is driven by a force oscillating at its eigenfrequency and the mechanical mode will start to oscillate coherently. The motion induces sidebands on the cavity field, such that photons with frequency ω_p are generated. These interfere destructively with the probe beam, leading to a dip in the transmission.

In the following, we focus on the single-photon strong coupling regime, where $g_0 \gtrsim \kappa$ and $g_0^2 \gtrsim \kappa\Omega$. In this regime, the impact of a *single* photon is already very large, such that we limit the discussion to weak laser driving first, i.e. $\alpha_p \ll \alpha_c \ll \kappa$.

Main OMIT dip – To compare against standard OMIT predictions, we first focus on the OMIT dip at resonance, as discussed above. Consider the level scheme of Fig. 2(b). Since both lasers are assumed to be weak, only the zero and one photon ladders are important for the following consideration. As above, we assume $\Delta_c = -\Omega$, such that the control beam hybridizes the states $|0, 1\rangle \leftrightarrow |1, 0\rangle$. This leads to a destructive interference of the two excitation pathways at $\Delta_p = 0$ and thus the OMIT dip.

The most important change in the OMIT signal for the single-photon strong coupling regime is due to the Franck-Condon factors. These also imply that the photon number $\langle \hat{a}^\dagger \hat{a} \rangle$ circulating inside the cavity is changed drastically [20, 26] and not given by $|\alpha|^2$ anymore. When replacing $|\alpha|^2$ by $\langle \hat{a}^\dagger \hat{a} \rangle$ in Eq. (7), however, we obtain quantitative agreement, cf. Fig. 2(c).

Note that this finding is not obvious a priori, since one might assume that the “granularity” of photons becomes important in the single-photon strong coupling regime [10, 19]. The shot noise in the control beam might lead to a stochastic variation in the depth of the OMIT dip, possibly leading to stronger effects. However, our studies show that this is not the case. This is because the spectral range of OMIT is set by $\sim \Gamma_M$. Thus, the probe beam transmission is controlled by the control beam intensity averaged over times Γ_M^{-1}. Crucially, this means that deterministic optical switching of the probe beam transmission using OMIT also works in the single-photon strong coupling regime, which we show now.

Let us now study how the system can be operated as an optomechanical transistor in a pulsed scheme, for large g_0. We consider an optomechanical cavity driven by the probe beam on resonance. Then, at $t = 0$, we switch on the control beam at the red mechanical sideband, $\Delta_c = -\Omega$, cf. Fig. 3(a). This leads to a decrease of the probe beam transmission $|\delta a|^2$ on a timescale set by $\sim \Gamma_M^{-1}$. When reaching the steady state, we switch off the control beam again, leading to a much more rapid increase of $|\delta a|^2$ on a scale $\sim \kappa^{-1}$, cf. Fig. 3(b). The influence of the control beam can also be seen in the population p_{n_a,n_b} of the states $|n_a, n_b\rangle$, cf. Fig. 3(c). The combination of control and probe first increases the population of the one-photon state $|1, 0\rangle$. Then, on a scale $\sim \Gamma_M^{-1}$, the population of the one-phonon state $|0, 1\rangle$ is increased, leading to a decreasing probe beam transmission. Furthermore, the two lasers exchange population of the zero and one photon ladder coherently, leading to oscillations at their beat frequency Ω.

Crossover from moderate to strong coupling – Let us now study what happens as a function of the “granular-

ity" or "quantum parameter" [10, 19] g_0/κ. For comparison, we will always keep $g_0 |\alpha|$ fixed while varying g_0, such that the linearized prediction is kept unchanged. It turns out that the most significant effects can be obtained when considering a resonant control beam, i.e. $\Delta_c = 0$ and observing the probe beam transmission close to the mechanical sidebands, i.e. $\Delta_p \approx n\Omega$, where n is an integer.

At the first mechanical sideband ($n = 1$), one observes a Fano resonance already at moderate coupling ($g_0/\kappa = 1/4$), cf. Fig. 4(a). This is because the probe beam probes both the first order, off-resonant $|1, 0\rangle \leftrightarrow |0, 0\rangle$ transition plus the second-order, resonant transition $|0, 1\rangle \leftrightarrow |0, 0\rangle$, the latter being a joint effect of the probe and control laser. Note that Fano resonances in general have recently been discussed in the context of optomechanics [33–37]. The shape of the resonance changes slightly upon increasing g_0/κ, as Franck-Condon factors become increasingly important [the leading order correction is $\mathcal{O}\big[g_0^2/\Omega^2\big]$].

Second sideband OMIT as a sensitive probe – Let us now consider the second mechanical sideband, $\Delta_p \approx 2\Omega$, while, again, $\Delta_c = 0$. The linearized analysis fails to show an OMIT signature at this sideband since it does not take into account transitions to sidebands $n > 1$. However, a full numerical simulation of (5) reveals that OMIT signatures do exist, which are thus a unique characteristic of the single-photon strong coupling regime in OMIT.

For moderate coupling $g_0 < \kappa$, the second mechanical sideband is too small to be observable in the transmission of the optomechanical cavity in the absence of two-tone driving. However, if the control beam is switched on, a significant feature develops in the OMIT signal, even for moderate coupling $g_0 = \kappa/4$, cf. Fig. 4(b). This feature is due to the two-photon transition $|0, 0\rangle \mapsto |1, 2\rangle \mapsto |0, 2\rangle$, with additional interference of other excitation pathways. Thus, we predict that a two-tone driving experiment should be able to identify signatures of single-photon strong coupling physics even for moderate coupling strengths $g_0 < \kappa \ll \Omega$. As we increase the coupling g_0 while keeping again $g_0 |\alpha|$ fixed, we find that this unique OMIT signal becomes even more pronounced. This is because the relevant higher order sideband transitions become more likely due to an increase of the Franck-Condon factors with g_0.

Since this second mechanical sideband shows unique signatures of the single-photon strong coupling regime even for moderate coupling (e.g. $g_0 = \kappa/4$), we further discuss what happens as we increase the probe driving strength α_p. We find that even for $\alpha_p \sim \alpha_c$ the normalized OMIT signal does not change. This is because the probe beam drives the cavity on the second sideband and thus the number of probe photons inside the cavity is still much lower than the number of control photons (as long as $\alpha_p/\alpha_c \ll \Omega/\kappa$). This is experimentally relevant, since one can therefore increase the absolute OMIT signal by increasing the probe intensity. Note that this also holds for the first mechanical sideband (but not for the main OMIT dip, where the normalized OMIT signal begins to be suppressed even for $\alpha_p/\alpha_c \sim \kappa/\Omega$).

We conclude that the challenge to bring out features of single-photon strong coupling physics in optomechanical systems will be greatly aided by two-tone driving.

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Note added: While preparing this manuscript for upload, two related works appeared on the arXiv [38, 39].

[1] T. J. Kippenberg and K. J. Vahala, Science **321**, 1172 (2008).
[2] F. Marquardt and S. M. Girvin, Physics **2**, 40 (2009).
[3] P. Meystre, arXiv:1210.3619 .
[4] M. Aspelmeyer, T. J. Kippenberg, and F. Marquardt, arXiv:1303.0733 .
[5] G. S. Agarwal and S. Huang, Phys. Rev. A **81**, 041803 (2010).
[6] S. Weis, R. Rivière, S. Deléglise, E. Gavartin, O. Arcizet, A. Schliesser, and T. J. Kippenberg, Science **330**, 1520 (2010).
[7] A. H. Safavi-Naeini, T. P. M. Alegre, J. Chan, M. Eichenfield, M. Winger, Q. Lin, J. T. Hill, D. E. Chang, and O. Painter, Nature **472**, 69 (2011).
[8] F. Massel, T. T. Heikkila, J.-M. Pirkkalainen, S. U. Cho, H. Saloniemi, P. J. Hakonen, and M. A. Sillanpaa, Nature **480**, 351 (2011).
[9] M. Fleischhauer, A. Imamoglu, and J. P. Marangos, Rev. Mod. Phys. **77**, 633 (2005).
[10] K. W. Murch, K. L. Moore, S. Gupta, and D. M. Stamper-Kurn, Nat Phys **4**, 561 (2008).
[11] F. Brennecke, S. Ritter, T. Donner, and T. Esslinger, Science **322**, 235 (2008).
[12] D. W. C. Brooks, T. Botter, S. Schreppler, T. P. Purdy, N. Brahms, and D. M. Stamper-Kurn, Nature **488**, 476 (2012).
[13] J. D. Teufel, T. Donner, D. Li, J. W. Harlow, M. S. Allman, K. Cicak, A. J. Sirois, J. D. Whittaker, K. W. Lehnert, and R. W. Simmonds, Nature **475**, 359 (2011).
[14] J. Chan, T. P. M. Alegre, A. H. Safavi-Naeini, J. T. Hill, A. Krause, S. Groblacher, M. Aspelmeyer, and O. Painter, Nature **478**, 89 (2011).
[15] E. Verhagen, S. Deleglise, S. Weis, A. Schliesser, and T. J. Kippenberg, Nature **482**, 63 (2012).
[16] J. Chan, A. H. Safavi-Naeini, J. T. Hill, S. Meenehan, and O. Painter, Applied Physics Letters **101**, 081115 (2012).
[17] S. Mancini, V. I. Man'ko, and P. Tombesi, Phys. Rev. A **55**, 3042 (1997).
[18] S. Bose, K. Jacobs, and P. L. Knight, Phys. Rev. A **56**,

4175 (1997).
[19] M. Ludwig, B. Kubala, and F. Marquardt, New Journal of Physics **10**, 095013 (2008).
[20] A. Nunnenkamp, K. Børkje, and S. M. Girvin, Phys. Rev. Lett. **107**, 063602 (2011).
[21] J. Qian, A. A. Clerk, K. Hammerer, and F. Marquardt, Phys. Rev. Lett. **109**, 253601 (2012).
[22] X.-W. Xu, H. Wang, J. Zhang, and Y. xi Liu, arXiv:1210.0070 .
[23] J. Li, S. Groeblacher, and M. Paternostro, arXiv:1211.5395 .
[24] A. Nunnenkamp, K. Børkje, and S. M. Girvin, Phys. Rev. A **85**, 051803 (2012).
[25] G.-F. Xu and C. K. Law, arXiv:1211.5445 .
[26] P. Rabl, Phys. Rev. Lett. **107**, 063601 (2011).
[27] A. Kronwald, M. Ludwig, and F. Marquardt, Phys. Rev. A **87**, 013847 (2013).
[28] K. Stannigel, P. Komar, S. J. M. Habraken, S. D. Bennett, M. D. Lukin, P. Zoller, and P. Rabl, Phys. Rev. Lett. **109**, 013603 (2012).
[29] M. Ludwig, A. H. Safavi-Naeini, O. Painter, and F. Marquardt, Phys. Rev. Lett. **109**, 063601 (2012).
[30] C. K. Law, Phys. Rev. A **51**, 2537 (1995).
[31] C. Gardiner and P. Zoller, *Quantum Noise* (Springer, 2004).
[32] A. Schließer, "Cavity optomechanics and optical frequency comb generation with silica whispering-gallery-mode microresonators," (2009).
[33] D. A. Rodrigues, Phys. Rev. Lett. **102**, 067202 (2009).
[34] F. Elste, S. M. Girvin, and A. A. Clerk, Phys. Rev. Lett. **102**, 207209 (2009).
[35] A. Xuereb, R. Schnabel, and K. Hammerer, Phys. Rev. Lett. **107**, 213604 (2011).
[36] T. Weiss, C. Bruder, and A. Nunnenkamp, arXiv:1211.7029 .
[37] K. Qu and G. S. Agarwal, arXiv:1304.0389 .
[38] K. Borkje, A. Nunnenkamp, J. D. Teufel, and S. M. Girvin, arXiv:1304.4155 .
[39] M.-A. Lemonde, N. Didier, and A. A. Clerk, arXiv:1304.4197 .